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# THE THEORY OF NON-LINEAR HARDENING OF AN ELASTOPLASTIC COMPOSITE MATERIAL FILLED WITH IDEALLY ELASTIC INCLUSIONS†

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A version of the statistical method of averaging the system of equilibrium equations for an elastoplastic two-component composite material in order to predict its macroscopic non-linear hardening is proposed. This version, unlike the averaging method developed previously in [1, 2], enables one to model and estimate the degree of connectedness of the matrix and the inclusions and to take into account the non-uniformity of the distribution and the development of plastic deformations. Macroscopic governing equations are constructed which describe the non-linear hardening of a composite material outside the elasticity limit, and its effective characteristics are calculated. © 1998 Elsevier Science Ltd. All rights reserved.

Suppose an isotropic two-component composite material, the elastoplastic matrix and elastic spherical inclusions of which are combined with ideal adhesion, occupies a volume $V$ bounded by a surface $S$. The mutual arrangement of the components in space is described by an indicator random isotropic function of the coordinates $\varkappa(\mathbf{r})$ (equal to zero at a set of points of the matrix $V_1$ and equal to unity at a set of points of the inclusions $V_2$), by means of which the local Hooke's law for the composite material considered can be written in the form

$$s_{ij}(\mathbf{r}) = 2\mu_1(e_{ij}(\mathbf{r}) - e_{ij}^p(\mathbf{r})) + 2[\mu]\varkappa(\mathbf{r})e_{ij}(\mathbf{r})$$

$$\sigma_{kk}(\mathbf{r}) = (3K_1 + 3[K]\varkappa(\mathbf{r}))\varepsilon_{kk}(\mathbf{r}) \tag{1}$$

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}, \quad e_{ij} = \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk}$$

Here $\sigma_{ij}$, $\varepsilon_{ij}$, $e_{ij}^p$ are the components of the stress, total and plastic strain tensors respectively, $\mu_s$, $K_s$ ($s = 1, 2$) are the shear moduli and the bulk moduli of the materials of the components and $[f] = f_2 - f_1$. The plastic strains satisfy the incompressibility condition $e_{kk}^p(\mathbf{r}) = 0$.

The function $\varkappa(\mathbf{r})$, the stresses, and the total and plastic strains are assumed to be random statistical homogeneous and ergodic fields, and their mathematical expectations are replaced by average values over the total volume $V$ and over the volumes of the components $V_s$ [3]

$$\langle f(\mathbf{r})\rangle = \frac{1}{V}\int_V f(\mathbf{r})d\mathbf{r}, \quad \langle f(\mathbf{r})\rangle_s = \frac{1}{V_s}\int_{V_s} f(\mathbf{r})d\mathbf{r}$$

The angle brackets denote the operation of averaging.

To determine the effective elasticity moduli of the composite material and to calculate the macroscopic residual strains, measured after the loads are removed from its surface, we must average the local equations (1) over the total volume $V$

$$\langle s_{ij}\rangle = 2\mu_1\langle e_{ij} - e_{ij}^p\rangle + 2[\mu]c_2\langle e_{ij}\rangle_2$$

$$\langle \sigma_{kk}\rangle = 3K_1\langle \varepsilon_{kk}\rangle + 3[K]c_2\langle \varepsilon_{kk}\rangle_2 \tag{2}$$

Here $c_2 = V_2V^{-1}$ is the volume content of the inclusions. The expressions on the right-hand sides of (2) show that to establish the effective Hooke's law we need to express the quantities $\langle \varepsilon_{ij}\rangle_2$ in terms of the macroscopic deformations $\langle \varepsilon_{ij}\rangle$ using the well-known relation [2]

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$$\langle \varepsilon_{ij} \rangle_2 = \langle \varepsilon_{ij} \rangle + c_2^{-1} \langle \varkappa' \varepsilon'_{ij} \rangle \tag{3}$$

The fluctuations of quantities in the volume $V$ are denoted by primes.

The fluctuations in the stress, total and plastic strain tensors can be found from Eqs (1) and (2)

$$s'_{ij}(\mathbf{r}) = 2\mu_1(e_{ij}(\mathbf{r}) - e^p_{ij}(\mathbf{r}))' + 2[\mu](\varkappa(\mathbf{r})e_{ij}(\mathbf{r}) - c_2\langle e_{ij}\rangle_2)$$
$$\sigma'_{kk}(\mathbf{r}) = 3K_1\varepsilon'_{kk}(\mathbf{r}) + 3[K](\varkappa(\mathbf{r})\varepsilon_{kk}(\mathbf{r}) - c_2\langle \varepsilon_{kk}\rangle_2)$$

The strained state of an elastic isolated inclusion when there are no plastic strains in the matrix is homogeneous and equal to its mean value $\langle\varepsilon_{ij}\rangle_2$ [3]. When the number of inclusions increases and plastic strains appear in the volume $V_1$ the homogeneity of the strain field in the volume $V_2$ is disturbed. Nevertheless, for a statistically homogeneous distribution of microspheres in the matrix their effect on one another balances out, and the strained state inside $V_2$ becomes close to homogeneous, but its value will, naturally, differ from the mean value. If we replace the tensor $\varkappa(\mathbf{r})\varepsilon_{ij}(\mathbf{r})$ by the quantity $\varkappa(\mathbf{r})\langle\varepsilon_{ij}\rangle_2$ approximately in the relations for the stress fluctuations, this assumption, depending on the sign of $[\mu]$ and $[K]$, reduces or increases the average density of the elastic potential in the volume $V$, which can be compensated by introducing an unknown parameter

$$\varkappa(\mathbf{r})\varepsilon_{ij}(\mathbf{r}) \cong \varkappa(\mathbf{r})\chi\langle\varepsilon_{ij}\rangle_2.$$

The relations for the stress fluctuations will then take the form

$$s_{ij}(\mathbf{r}) = 2\mu_1(e_{ij}(\mathbf{r}) - e^p_{ij}(\mathbf{r})) + 2[\mu]\varkappa(\mathbf{r})\chi\langle e_{ij}\rangle_2$$
$$\sigma_{kk}(\mathbf{r}) = 3K_1\varepsilon_{kk}(\mathbf{r}) + 3[K]\varkappa(\mathbf{r})\chi\langle\varepsilon_{kk}\rangle_2 \tag{4}$$

The coefficient $\chi$, characterizing the difference between the strained state of the inclusion and its mathematical expectation, describes the interaction of the inclusions with one another (the connectedness of the components).

To calculate the moments $\langle\varkappa'\varepsilon'_{ij}\rangle$ we must supplement the following equilibrium equations to relations (4)

$$\sigma_{ip,p}(\mathbf{r}) = 0 \tag{5}$$

and the Cauchy formulae

$$2\varepsilon_{ij}(\mathbf{r}) = u_{i,j}(\mathbf{r}) + u_{j,i}(\mathbf{r}) \tag{6}$$

which relate the components of the strain tensor with the components of the displacement vector $u_i(r)$. The system of equations (4)–(6) are closed with respect to the stresses $\sigma_{ij}$, the total strains $\varepsilon_{ij}$ and the displacements $u_i$, and the plastic strains $\varepsilon^p_{ij}$ are parameters (it is assumed that the loading history is known in each specific problem). The conditions for all the quantities on the surface of the volume $V$ to be homogeneous are the boundary conditions for the system obtained, namely

$$f(\mathbf{r})|_{\mathbf{r}\in S} = \langle f\rangle$$

Using Green's tensor $G_{ik}(r)$ we can replace the system of equations (4)–(6) by the following system of integral equations [2]

$$\varepsilon'_{ij}(\mathbf{r}) = \int_V G_{ik,lj}(\mathbf{r} - \mathbf{r}_1)(2\mu_1\varepsilon^{p'}_{kl}(\mathbf{r}_1) + \chi\langle\tau_{kl}\rangle_2\varkappa'(\mathbf{r}_1))d\mathbf{r}_1 \tag{7}$$

$$\tau_{kl} = -2[\mu]e_{kl} - \delta_{kl}3[K]\varepsilon_{pp}$$

Multiplying both sides of Eqs (7) by $\varkappa'(\mathbf{r})$ and averaging over the total volume $V$ we obtain

$$\langle\varkappa'\varepsilon'_{ij}\rangle = -c_2\alpha_{ij} + c_1c_2(\alpha_1\langle\tau_{ij}\rangle_2 - \beta_1\delta_{ij}\langle\tau_{pp}\rangle_2)/2\mu_1 \tag{8}$$

$$\alpha_1 = \frac{2(4-5\nu_1)\chi}{15(1-\nu_1)}, \quad \beta_1 = \frac{\chi}{15(1-\nu_1)}, \quad \nu_1 = \frac{3K_1 - 2\mu_1}{6K_1 + 2\mu_1}$$

$$\alpha_{ij} = -\frac{2\mu_1}{c_2}\left\langle \int_V G_{ik,lj}(r - r_1)\varkappa'(r)e^{p'}_{kl}(r_1)dr_1 \right\rangle$$

Here $\alpha_{ij}$ are the components of the tensor characterizing the inhomogeneous distribution and the development of structural plastic strains in the matrix, and $c_1 = V_1V^{-1}$ is the volume content of the matrix.

Substituting (3) and (8) into (2) and separating the deviator and volume parts we obtain the effective Hooke's law for the composite material in question

$$\langle s_{ij}\rangle = 2\mu^*(\langle e_{ij}\rangle - e^*_{ij}),\ \ \langle\sigma_{pp}\rangle = 3K^*(\langle\varepsilon_{pp}\rangle - \varepsilon^*_{pp}) \tag{9}$$

$$\mu^* = \mu_1\left(1 + \frac{c_2(m-1)}{1 + c_1\alpha_1(m-1)}\right)$$

$$K^* = K_1\left(1 + \frac{c_2(q-1)}{1 + c_1\gamma_1(q-1)}\right)$$

$$m = \frac{\mu_2}{\mu_1},\ \ q = \frac{K_2}{K_1},\ \ \gamma_1 = \frac{(1+\nu_1)\chi}{3(1-\nu_1)}$$

Here $\mu^*, K^*$ are the effective shear and bulk moduli of the composite material, and $\varepsilon^*_{ij}$ are the residual strains, which are measured after the loads are removed from the surface of the volume $V$. The connection between the residual and plastic strains is found from relations (2), (3) and (8) if we put $\langle\sigma_i\rangle = 0$, $\langle\varepsilon_{ij}\rangle = \varepsilon^*_{ij}$ in them

$$\langle e^p_{ij}\rangle = m^*(e^*_{ij} - a_{ij}) + a_{ij},\ \ \alpha_{pp} = q^*/(q^*-1)\varepsilon^*_{pp} \tag{10}$$

$$m^* = \mu^*/\mu_1;\ \ q^* = K^*/K_1;\ \ a_{ij} = \alpha_{ij} - \tfrac{1}{3}\delta_{ij}\alpha_{pp}.$$

The expressions for the effective elasticity moduli $\mu^*$ and $K^*$ depend very much on the connectivity parameter $\chi$. When $\chi = 1$ we obtain the Kerner model for a single microsphere. In addition, depending on the sign of the quantities $[\mu]$ and $[K]$ this model is identical either with the upper or lower Hashin–Shtrikman boundaries [3]. The limiting value of $\chi = 0$ corresponds to the maximum connectedness of the components for which the effective moduli are identical with the mean values (the upper Voight limit): $\mu^* = \mu_1c_1 + \mu_2c_2$, $K^* = K_1c_1 + K_2c_2$.

Relations (9) and (10) show that despite the plastic incompressibility of the matrix material, the composite material as a whole acquires a qualitatively new property, namely, a certain irreversible compressibility, which does not have constituent components separately. This compressibility is due to the different values of the bulk moduli $K_1$ and $K_2$ and only disappears when $K_1 = K_2$.

Suppose the plastic properties of the matrix material are specified by the Mises yield surface $s_{ij}(\mathbf{r})s_{ij}(\mathbf{r}) = k_2$, $\mathbf{r} \in V_1$ and associated plastic flow law

$$s_{ij}(\mathbf{r}) = k\frac{\dot{e}^p_{ij}(\mathbf{r})}{\sqrt{\dot{e}^p_{kl}(\mathbf{r})\dot{e}^p_{kl}(\mathbf{r})}},\ \ \mathbf{r} \in V_1 \tag{11}$$

Here $k$ is the yield point of the matrix material for shear and $\dot{e}^p_{ij}$ are the plastic strain rates.

In order to use the above averaging method further it is necessary to linearize the non-linear local equation (11) by making certain assumptions. Following the procedure described previously [4], we will neglect fluctuations of the invariant of the tensor of the plastic strain rates within the volume of the matrix $V_1$. Equation (11) then takes the form

$$s_{ij}(\mathbf{r}) = \frac{k}{\Lambda}\dot{e}^p_{ij},\ \ \mathbf{r} \in V_1,\ \ \Lambda = \sqrt{\langle\dot{e}^p_{kl}\dot{e}^p_{kl}\rangle_1} \tag{12}$$

It was shown in [4] that the above assumption increases the average energy dissipation density and, for certain types of structures, leads to upper limits on the effective constants of the composite medium.

Taking into account the fact that under active loading conditions in the composite material outside the elasticity limit, the quantity $\Lambda$ is always positive, we make the following replacement of the variable

$t$: $d\tau = \Lambda dt$. Relation (12) then takes the form

$$s_{ij} = kde_{ij}^p / d\tau \tag{13}$$

Substituting Hooke's law for the material of the first component into Eq. (13), we obtain

$$kde_{ij}^p / d\tau = 2\mu_1(e_{ij}(\mathbf{r}) - e_{ij}^p(\mathbf{r})), \quad \mathbf{r} \in V_1 \tag{14}$$

Multiplying both sides of Eq. (14) by the quantity

$$-2\mu_1 c_2^{-1} G_{ik,lj}(\mathbf{r} - \mathbf{r}_1)\varkappa'(\mathbf{r})d\mathbf{r}_1$$

integrating them in the region $V$ with respect to the variables $\mathbf{r}_1$ and $\mathbf{r}$ and using the properties of the second derivative of Green's tensor, we obtain [2]

$$kd\alpha_{ij} / d\tau = 2\mu_1\gamma_1(\langle\varepsilon_{ij}\rangle_2 - \langle\varepsilon_{ij}\rangle) + 2[\mu]c_1\alpha_1\langle e_{ij}\rangle_2 + \delta_{ij}[K]c_1(\alpha_1 - 3\beta_1)\langle\varepsilon_{pp}\rangle_2$$

Separating the deviator part and the volume part and also taking Eqs (7) and (9) and relation (3) into account, we obtain

$$\begin{aligned}
&m^*k\frac{da_{ij}}{d\tau} = \xi\langle s_{ij}\rangle + 2\mu^*\xi e_{ij}^* - 2\mu^*(\xi + \gamma_1)a_{ij} \\
&\frac{q^*k}{\alpha_1 - 3\beta_1}\frac{d\varepsilon_{pp}^*}{d\tau} = (q^* - 1)\left(\frac{qw}{q^*} - 1\right)\langle\sigma_{pp}\rangle - 3K_2 w\varepsilon_{pp}^* \\
&\xi = c_1\alpha_1(m - 1)(1 - \gamma_1)/(1 + c_1\alpha_1(m - 1)) \\
&w = 1/(1 + c_1\gamma_1(q - 1))
\end{aligned} \tag{15}$$

Since at any point of the composite material, both for active loading and unloading, the stresses $s_{ij}(r)$ do not exceed the amounts $kde_{ij}^p(\mathbf{r})/d\tau$, we have the following inequalities

$$\langle s_{ij}\rangle_s \leqslant kd\langle e_{ij}^p\rangle_1 / d\tau \quad (s = 1, 2)$$

from which the following upper limit of the macroscopic associated flow law follows

$$\langle s_{ij}\rangle = kc_1^{-1}d\langle e_{ij}^p\rangle / d\tau \tag{16}$$

Eliminating the components of the tensor $\langle e_{ij}^p\rangle$, $a_{ij}$ from relations (9), (15) and (16), we obtain the following effective governing equations of the composite material outside the elasticity limit

$$\begin{aligned}
&p_1\frac{d\langle s_{ij}\rangle}{d\tau} + p_0\langle s_{ij}\rangle = q_1\frac{de_{ij}^*}{d\tau} + q_2\frac{d^2e_{ij}^*}{d\tau^2} \\
&\langle\sigma_{pp}\rangle = n_0\varepsilon_{pp}^* + n_1\frac{d\varepsilon_{pp}^*}{d\tau}
\end{aligned} \tag{17}$$

Here

$$\begin{aligned}
&p_0 = 2\mu^*c_1(\xi + \gamma_1)k^{-1}, \quad p_1 = m^*c_1 + \xi(m^* - 1) \\
&q_1 = 2\mu^*(\gamma_1 m^* + \xi), \quad q_2 = km^{*2}, \quad n_0 = \frac{3K_2wq^*}{(q^* - 1)(qw - q^*)} \\
&n_1 = \frac{kq^{*2}}{(\alpha_1 - 3\beta_1)(q^* - 1)(qw - q^*)}
\end{aligned}$$

The equations of non-linear hardening of the composite material outside the elasticity limit (17) contain the unknown connectivity parameter of the components $\chi$. It can be calculated from the equations of elastic strains of the medium (9) if we take into account the well-known experimental value of one of the effective elasticity moduli of the composite material, for example, Young's modulus

$$\frac{9K^*\mu^*}{3K^*+2\mu^*}=E^*_{\exp} \tag{18}$$

The value of $\chi$ thus obtained can henceforth be used in the non-linear equations (17), since for small strains the structure of the composite material is not changed to any considerable extent.

The equations of non-linear hardening (17) must be supplemented by the equations of the initial and limit yield surfaces. Since the inclusions are always in an elastic state and are stress concentrators, we can assume that plastic flow begins in the matrix in the region of the surface of the inclusions, when the deviator components of the stress tensor in the region $V_2$ reach the yield point of the matrix: $\langle s_{ij}\rangle_2\langle s_{ij}\rangle_2 = k^2$ or taking Hooke's law into account $4\mu_2\langle e_{ij}\rangle_2\langle e_{ij}\rangle_2 = k^2$. Hence, substituting expressions $\langle e_{ij}\rangle_2 = (1+c_1\alpha_1(m-1))^{-1}\langle e_{ij}\rangle$ and using the effective Hooke's law (9) we obtain the initial macroscopic surface and yield point

$$\langle s_{ij}\rangle\langle s_{ij}\rangle = k_0^{*2},\quad k_0^* = km^{-1}(1+(m-1)(c_1\alpha_1+c_2)) \tag{19}$$

The limit yield surface of the composite material corresponds to plastic strains which considerably exceed the elastic strains, and hence we can neglect the elastic strains when determining the maximum effective yield point. Obtaining the limit yield surface of the composite then reduces to solving the rigid-plastic problem for a composite material with absolutely rigid inclusions. The relation between the stresses and strain rates in this case is given by the equation [4]

$$s_{ij}(\mathbf{r}) = k\Lambda^{-1}\dot{e}_{ij}(\mathbf{r}) + \langle s_{ij}\rangle_2\varkappa(\mathbf{r}) \tag{20}$$

The macroscopic stresses are obtained after averaging (20) over the total volume $V$

$$\langle s_{ij}\rangle = k\Lambda^{-1}\langle\dot{e}_{ij}\rangle + c_2\langle s_{ij}\rangle_2 \tag{21}$$

The fluctuations of the stress–strain state, taking the connectivity parameter $\chi$ into account, are related by the equation

$$s'_{ij}(\mathbf{r}) = \frac{k}{\Lambda}\dot{e}'_{ij}(\mathbf{r}) + \varkappa'(\mathbf{r})\chi\langle s_{ij}\rangle_2 \tag{22}$$

Applying the above procedure for determining the effective properties of the composite material (5)–(8) together with (21) to Eq. (22), we obtain

$$\langle s_{ij}\rangle = \frac{k}{\Lambda}\frac{5-c_1(5-2\chi)}{2c_1\chi}\langle\dot{e}_{ij}\rangle \tag{23}$$

The quantity $\Lambda$ is found from the well-known relation for the average energy dissipation density [4]

$$\langle D\rangle = \langle s_{ij}\rangle\langle\dot{e}_{ij}\rangle = k\Lambda \tag{24}$$

Eliminating $\Lambda$ from relations (23) and (24) we obtain the macroscopic limit yield surface and the corresponding effective yield point

$$\langle s_{ij}\rangle\langle s_{ij}\rangle = k_\infty^2,\quad k_\infty = k\left[\frac{5-c_1(5-2\chi)}{2c_1\chi}\right]^{1/2} \tag{25}$$

In particular, when $\chi = 1$, Eqs (25) are identical with the analogous results obtained in [4].

We will use (17), (19) and (25) to calculate the non-linear hardening of a composite material in the case of simple uniaxial loading ($\sigma_1 \neq 0$, $\sigma_2 = \sigma_3 = 0$). We have for the principal strain values $\varepsilon_1^* \neq 0$, $\varepsilon_2^* = \varepsilon_3^*$. In the case of simple loading the residual strains obey the law

$$\varepsilon_1^* = \varepsilon_1^0 \tau, \quad \varepsilon_1^0 = \text{const} \tag{26}$$

Relations (17) then take the form

$$p_1 \frac{d\sigma_1}{d\tau} + p_0 \sigma_1 = q_1 \left( \varepsilon_1^0 - \frac{d\varepsilon_2^*}{d\tau} \right) - q_2 \frac{d^2 \varepsilon_2^*}{d\tau^2}$$

$$\sigma_1 = n_0 (\varepsilon_1^0 \tau + 2\varepsilon_2^*) + n_1 \left( \varepsilon_1^0 + 2 \frac{d\varepsilon_2^*}{d\tau} \right) \tag{27}$$

Eliminating $\sigma_1$ from (27) we obtain a second-order differential equation for $\varepsilon_2^*$ with the following boundary conditions

$$\frac{d^2 \varepsilon_2^*}{d\tau^2} + a_1 \frac{d\varepsilon_2^*}{d\tau} + a_0 \varepsilon_2^* = f_0 \tag{28}$$

$$\varepsilon_2^*(0) = 0, \quad \sigma_1(0) = \sigma_0, \quad \sigma_1(\infty) = \sigma_\infty$$

Here

$$a_1 = b_1 + 2b_2, \quad a_0 = \frac{2p_0 n_0}{2p_1 n_1 + q_2}$$

$$b_1 = \frac{q_1}{2p_1 n_1 + q_2}, \quad b_2 = \frac{p_1 n_0 + p_0 n_1}{2p_1 n_1 + q_2}$$

$$f_0 = -\frac{a_0}{2} \varepsilon_1^0 \tau + (b_1 - b_2) \varepsilon_1^0, \quad \sigma_0 = \sqrt{3/2} k_0^*, \quad \sigma_\infty = \sqrt{3/2} k_\infty^*$$

Solving (27) and (28) we obtain

$$\varepsilon_1^* = \frac{2p_0}{3q_1} \sigma_\infty \tau, \quad \varepsilon_2^* = \frac{\sigma_\infty}{2n_0} - \frac{p_0}{3q_1} \sigma_\infty \tau + \xi_+(\tau) + \xi_-(\tau) \tag{29}$$

$$\sigma_1 = \sigma_\infty + \eta_+(\tau) + \eta_-(\tau)$$

Here

$$\xi_\pm(\tau) = (2n_0 n_1 (r_\pm - r_\mp))^{-1} (\sigma_0 n_0 + n_1 \sigma_\infty r_\mp) \exp(r_\pm \tau)$$

$$\eta_\pm(\tau) = 2(n_0 + n_1 r_\pm) \xi_\pm(\tau)$$

and $r_\pm = -a_1/2 \pm \sqrt{((a_1/2)^2 - a_0)}$ are the roots of the characteristic equation of differential equation (28).

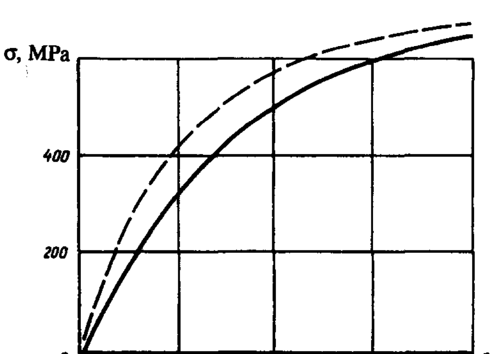

Fig. 1.

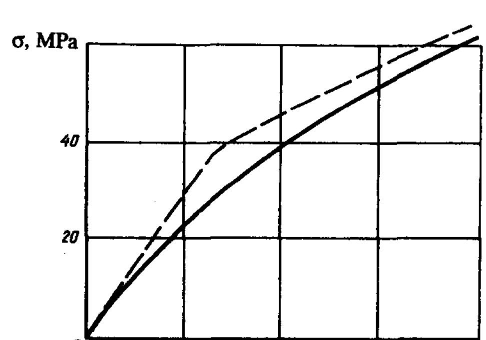

Fig. 2.

Eliminating $\tau$ from (29) we obtain the following relation between the transverse and longitudinal residual strains

$$\varepsilon_2^* = -\nu^*(\varepsilon_1^*)\varepsilon_1^* \tag{30}$$

and the law of uniaxial non-linear hardening of the composite material outside the elasticity limit

$$\sigma_1 = E^*(\varepsilon_1^*)\varepsilon_1^* \tag{31}$$

Here

$$\nu^*(\varepsilon_1^*) = \tfrac{1}{2} - (\sigma_\infty/(2n_0) + \zeta_+(\varepsilon_1^*) + \zeta_-(\varepsilon_1^*))/\varepsilon_1^*$$

$$E^*(\varepsilon_1^*) = (\sigma_\infty + \psi_+(\varepsilon_1^*) + \psi_-(\varepsilon_1^*))/\varepsilon_1^*$$

$$\zeta_\pm(\varepsilon_1^*) = (2n_0n_1(r_\pm - r_\mp))^{-1}(\sigma_0 n_0 + n_1\sigma_\infty r_\mp)\exp\left(\frac{3r_\pm q_1\varepsilon_1^*}{2p_0\sigma_\infty}\right)$$

$$\psi_\pm(\varepsilon_1^*) = 2(n_0 + n_1 r_\pm)\zeta_\pm(\varepsilon_1^*)$$

($\nu^*(\varepsilon_1^*)$ is the elastoplastic Poisson's ratio and $E^*(\varepsilon_1^*)$ is the modulus of tensile (compressive) plasticity).

Figure 1 compares the theoretical diagrams of uniaxial tension of samples of a composite material based on a copper matrix and a baked framework of tungsten powder (the dashed curve), calculated from (9), (18), (30) and (31), with experimental data given in [5] (the continuous curve). The calculated values of the mechanical characteristics are as follows: $E_1 = 1.12 \times 10^5$ MPa, $E_2 = 3.6 \times 10^5$ MPa, $\nu_1 = 0.369$, $\nu_2 = 0.2$, $c_2 = 0.66$, $k = 250$ MPa, $E^*_{\exp} = 2.46 \times 10^5$ MPa, and the calculated value $\varkappa = 1.406$.

From (9), (18), (30) and (31) we also calculated the diagram of uniaxial tension for an epoxy matrix, hardened with glass microspheres. The tensile diagram of the material of the matrix was approximated by a piecewise diagram of an ideally elastoplastic solid. The calculated values of the quantities were as follows: $E_1 = 0.307 \times 10^4$ MPa, $E_2 = 7.35 \times 10^4$ MPa, $\nu_1 = 0.45$, $\nu_2 = 0.21$, $c_2 = 0.76$, $k = 69$ MPa, $E_{exp} = 0.613 \times 10^4$ MPa, and the calculated value $\varkappa = 0.659$.

Figure 2 compares the theoretical diagram (the dashed curve) with the experimental diagram (the continuous curve), drawn using the data in [6].

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